Peer-to-Peer Energy Trading in Smart Grid Considering Power Losses and Network Fees

Amrit Paudel, *Graduate Student Member, IEEE*, L. P. M. I. Sampath, *Member, IEEE*, Jiawei Yang, *Graduate Student Member, IEEE*, and Hoay Beng Gooi, *Life Senior Member, IEEE*

***Abstract*—Peer-to-peer (P2P) energy trading is one of the promising approaches for implementing decentralized electricity market paradigms. In the P2P trading, each actor negotiates directly with a set of trading partners. Since the physical network or grid is used for energy transfer, power losses are inevitable, and grid-related costs always occur during the P2P trading. A proper market clearing mechanism is required for the P2P energy trading between different producers and consumers. This paper proposes a decentralized market clearing mechanism for the P2P energy trading considering the privacy of the agents, power losses as well as the utilization fees for using the third party owned network. Grid-related costs in the P2P energy trading are considered by calculating the network utilization fees using an electrical distance approach. The simulation results are presented to verify the effectiveness of the proposed decentralized approach for market clearing in P2P energy trading.**

***Index Terms*—Peer-to-peer energy trading, network utilization fees, market clearing, decentralized approach.**

NOMENCLATURE

$[x]^+$	$\max(0, x)$.
β_j, θ_j	Utility function parameters of consumer j.
γ	Network usage charge per unit electrical distance.
κ	Step size for Lagrangian multiplier update.
λ_i	Per unit price of energy from producer i.
$\mathbf{B}$	Admittance matrix of order $N \times N$.
$\mathbf{H}$	Network matrix of order $L \times N$.
$\mathcal{L}$	Set of lines in the network.
$\underline{p}_i/\overline{p}_i$	Minimum/maximum generation of producer i.
$\underline{p}_j/\overline{p}_j$	Minimum/maximum demand of consumer j.
ϱ_i	Coefficient of losses for producer i.
a_i, b_i, c_i	Cost function parameters of producer i.
d_{ji}	PTD between consumer j and producer i.
k	Index of iterations.
l	Index of lines.
N	Total number of agents/nodes.
n_c	Total number of consumers.
n_p	Total number of producers.
p_i	Total generation of producer i.
p_j	Total demand of consumer j.
p_{ji}	Power demand of consumer j from producer i.
TF_j	Total network utilization fee for consumer j.
$U_j(\cdot)$	Utility function of consumer j.
W_i	Total welfare of producer i.
W_j	Total welfare of consumer j.
w_{ji}	Welfare of consumer j from the trading with producer i.

Manuscript received December 2, 2019; revised April 13, 2020 and May 17, 2020; accepted May 24, 2020. Date of publication May 27, 2020; date of current version October 21, 2020. This work was supported by the Agency for Science, Technology and Research (A*STAR) through its Singapore–Germany Academic-Industry (2+2) International Collaboration under Grant A1990b0060. Paper no. TSG-01805-2019. *(Corresponding author: Amrit Paudel.)*

The authors are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore (e-mail: amrit003@e.ntu.edu.sg).

Color versions of one or more of the figures in this article are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSG.2020.2997956

I. INTRODUCTION

A. Background and Motivation

THE INCREASING presence of more proactive actors or agents in the current power system has triggered a design and adaptation of a more decentralized paradigm to power systems and electricity market operation [1], [2]. Peer-to-peer (P2P) energy trading is one of the promising approaches for implementing decentralized electricity market paradigms. In the P2P trading, each actor negotiates directly with a set of trading partners without any intervention of a conventional intermediary [3]. A P2P market platform enables direct energy transactions among producers and consumers in the electricity network [4]. Since the P2P energy trading in smart grids is a new concept, a proper market clearing mechanism is required for the P2P energy trading between different producers and consumers [5]. A market clearing mechanism deals with electricity pricing and energy allocation. The market clearing method should be computationally efficient and set with a defined trading objective. The objective of the trading should be designed in such a way that it incentivizes the participation of agents in the P2P market. Besides, agents in the market behave independently with their interest and have a set of private information that they do not want to reveal. Therefore, designing a proper market clearing mechanism for P2P energy trading while maintaining privacy is a challenging task.

On the other hand, a physical network or grid, which is used for energy transfer, imposes various grid-related aspects on energy trading. Such networks or grids usually owned by third parties other than agents participating in the market. Grid-related costs always occur during each energy trade. They mainly account for investment costs to build a network as well as operation and maintenance expenses [6]. In real applications, the owner of the grid collects the network utilization fees, paid by consumers corresponding to its network usage during energy trade, to cover these costs. In traditional electricity markets, such network fees constitute a significant portion of the energy bills of consumers [7]. However, the P2P energy trading enables consumers to reduce network utilization fees by strategically choosing producers. To achieve this objective, a factor related to network usage should be considered while designing a market clearing mechanism for the P2P trading. Besides, the power losses in the network lines are inevitable during the power transmission process in P2P energy trading. Therefore, it is essential to consider power losses in the P2P trading model to make it more practical. In summary, power losses in the network and grid-related costs should be properly integrated into the P2P trading model. Hence, developing a privacy-preserving and fair market clearing mechanism for the P2P energy trading considering various aspects of the grid is the motive behind this work.

B. Related Works and Contributions

In the literature, there has been an increasing interest in the area of market design for P2P energy trading. Khorasany *et al.* [1], has presented a decentralized bilateral energy trading system for P2P electricity markets. A P2P energy trading under network constraints is proposed in [3] and the impact of P2P transactions on the network is assessed by sensitivity analysis to ensure the exchange of energy does not violate the network constraints. A game theory based pricing model for the P2P energy trading is proposed in [5] to maximize the social welfare of the buyers and sellers in a prosumer based community microgrid. P2P negotiations are initiated with the help of a coordinating agent called P2P market operator. In [8], a coordinated market model for P2P energy trading and ancillary services in distribution grids is proposed. The distribution network operator manages ancillary services in the network during P2P energy transactions. A P2P energy trading in virtual microgrids with heterogeneous prosumers is proposed in [7], where interactions among the prosumers are modeled as a non-cooperative game.

An indirect customer-to-customer (iC2C) energy trading in the distribution level is proposed by Chen and Su in [9]. Agents update their trading strategies using the reinforcement learning principle, and an energy broker acts as a coordinator for managing the market operations. Suppliers act as intermediaries in a forward bilateral contract network designed for the real-time P2P energy trading between sellers and buyers in [10]. Tushar *et al.* [11], has presented a P2P energy trading mechanism where a centralized power system decides the energy price to incentivize the prosumers to participate in P2P energy trading to reduce the peak demand on the centralized power system. A novel P2P model for joint trading of energy and uncertainty in the local electricity market is proposed in [12]. Liu *et al.* [13], has proposed a double auction-based P2P energy trading for residential demand response to face disturbances. A consensus-based energy management scheme for smart grids is proposed in [14]–[16], where the coordination among the agents through only local information exchange among neighbors establishes the supply-demand balance.

Most of the existing studies on P2P energy trading often neglect the power losses in the network, assuming that the energy is transmitted over a short distance in the distribution system. However, without considering power losses, power flows of P2P trading decisions cannot satisfy the power balance condition for the stable operation of the power system, and the practicability of the P2P trading becomes questionable. Since the power losses occurring in the network directly impact the market outcome, a transparent loss allocation framework is required to ensure the economic fairness among agents in the P2P market. In [17], network losses in a microgrid are allocated to each node and compensated by discharging battery storage units at the corresponding node. Kim and Dvorkin [18], has proposed a P2P market considering power losses, where costs are allocated to each trade based upon its grid usage. A graph-based loss allocation framework is proposed in [19] for transactive energy markets in distribution systems. In [20], the bilateral exchange coefficient is used to calculate the losses cost associated with each P2P transaction between nodes.

In these works [1], [3], [5], [7], [9], [10], [11], [12], [13], different approaches for P2P energy trading are studied considering the economic aspect for increasing welfare or decreasing the cost of agents. However, the grid-related costs of using the third party owned network for power distribution as well as power losses in P2P energy transactions are not considered. In addition, [5], [8]–[11] consider a separate entity to coordinate the energy trading. The coordinator has different roles, such as initiate the energy trading [5]; manage the network services [8]; and perform the market clearing [9]–[11], based on the market model. The vital role of the coordinator in market clearing endangers agents' privacy as well as the system scalability of the market as all agents need to communicate with the coordinator responsible for market clearing and share sufficient information with it. The role of the coordinator in market clearing is eliminated using a consensus-based approach to enhance the privacy of the agents in [14]–[16], but there is no direct negotiation among agents. The direct negotiation among agents is the rationale behind P2P trading. Also, the grid-related aspects are not considered.

On the other hand, [17]–[20] propose different approaches for losses and the associated cost allocation, but have not studied their impact on the market outcome. The proposed P2P markets in [18]–[20] rely on the distribution network operator for the compensation of losses. This increases the dependency of the local P2P market on the upstream market. The incurred losses caused by each transaction in the local market should be compensated to achieve a feasible market solution. Therefore, the incurred losses should be included in the market model and compensated within the local market itself to increase the autonomy of the P2P market. Besides, all agents behave greedily and always try to reach the optimal solution for themselves. The market clearing mechanism should be designed in such a way that the optimal local solution should coincide with an optimal global solution, and it should be fair to all agents.

To this end, there is a lack of a proper framework for the P2P energy trading, which considers the network usage during P2P energy transactions along with the privacy of agents. The focus of this paper is on the design of a proper market clearing mechanism for P2P energy trading, considering the privacy of

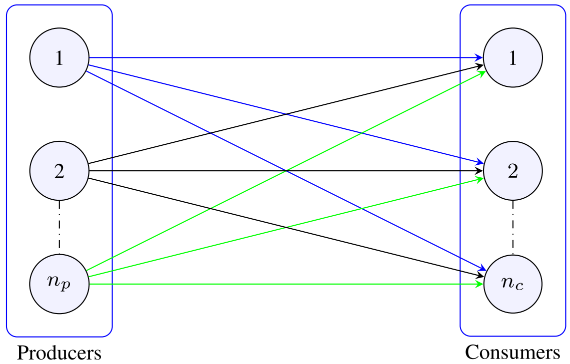

Fig. 1. The schematic model of P2P energy market.

the agents, power losses as well as the utilization fees for using the third party owned network during energy trade. This paper proposes a fully decentralized approach for market clearing in the P2P energy market. The proposed approach uses iterative negotiations and local decision making to eliminate the need of the coordinator for market clearing. In addition, power losses and network utilization fees during P2P energy trading are taken into account. The novel contributions made in this paper are as follows:

1) A P2P energy trading is formulated as a social welfare maximization problem with consideration of network/grid usage, i.e., power losses and network utilization fees, in P2P energy transactions.
2) An electrical distance approach is proposed to calculate the network fees for P2P energy trade. The network fees are proportional to the electrical distance between producers and consumers. In this approach, longer distance transactions become more expensive because of higher network fees, so consumers are encouraged to trade with producers at a shorter electrical distance.
3) A novel decentralized approach, which neither requires any third-party nor reveals any private information of the agents, is proposed for market clearing in the P2P energy trading. The proposed decentralized algorithm solves the P2P market clearing problem without sharing the agents' preferences and respects the privacy of agents.

C. Paper Organization

In Section II, detailed problem formulation for the P2P energy trading is explained. A decentralized approach for market clearing in the P2P energy trading is discussed in Section III. Simulation results are illustrated in Section IV, and conclusions are drawn in Section V.

II. Problem Formulation

Consider a smart grid consisting of a set $\mathcal{P} \triangleq \{1, \dots, n_p\}$ of producers with index $i \in \mathcal{P}$ and a set $\mathcal{C} \triangleq \{1, \dots, n_c\}$ of consumers with index $j \in \mathcal{C}$. The set of all agents is $\mathcal{N} = \mathcal{P} \cup \mathcal{C}$, and $\mathcal{P} \cap \mathcal{C} = \phi$. The total number of agents is $N \triangleq n_p + n_c$. The smart grid has the electrical network and the communication network. The electrical network is used for energy transfer and the communication network is used for information exchange among agents. Smart meters are installed at the premise of each agent. All the communication tasks are done through the smart meters using communication infrastructure. The envisioned electricity market for the P2P energy trading in a smart grid consists of multiple producers and multiple consumers, as shown in Fig. 1. The producers and consumers have flexible production and consumption, respectively. The market is cleared through P2P interactions among producers and consumers. In this paper, we focus on the market clearing for a single market period of one hour in the P2P energy market.[1] The terms power and energy are used interchangeably since the market period considered is one hour.

Since the proposed electricity market has n_p producers and n_c consumers, every possible bilateral trades can be condensed in a demand matrix $\mathbf{P} \in \mathbb{R}^{n_c \times n_p}$ as in (1). Each element p_{ji} of the demand matrix $\mathbf{P}$ represents the power demand of consumer j from producer i. All entries of the demand matrix are considered as the decision variables.

$$\mathbf{P} = \begin{bmatrix} p_{11} & \cdots & p_{1n_p} \\ \vdots & \ddots & \vdots \\ p_{n_c1} & \cdots & p_{n_cn_p} \end{bmatrix} \tag{1}$$

The j^{th} row of $\mathbf{P}$, denoted by vector $\mathbf{P_j} \in \mathbb{R}^{1 \times n_p}$ gives the demand schedule of consumer j. The total demand of consumer j is given by

$$p_j = \sum_{i \in \mathcal{P}} p_{ji} \tag{2}$$

Similarly, the i^{th} column of $\mathbf{P}$, denoted by $\mathbf{P_i} \in \mathbb{R}^{n_c \times 1}$ gives the supply schedule of producer i. For each producer i

$$p_i - \phi_i(p_i) = \sum_{j \in \mathcal{C}} p_{ji} \tag{3}$$

In (3), $\phi_i(p_i)$ is the power losses induced by producer i. The power losses are separable and can be approximated as a nonlinear function of p_i as follows [21]:

$$\phi_i(p_i) = \varrho_i p_i^2 \tag{4}$$

The value of loss-coefficient ϱ_i depends on the parameters and configuration of the the network model [21].

A. Consumer and Producer Model

The responses of different consumers to various scenarios can be modeled by using the concept of the *utility function* [22]. The utility function represents the personal satisfaction or convenience for electricity usage, and it can be expressed as a function of energy demand. The utility function of consumer j is denoted by $U_j(p_j)$, and it should have the following properties:

- $U_j'(p_j) \geq 0$, i.e., it is a non-decreasing function.
- $U_j''(p_j) \leq 0$, i.e., utility will get saturated.
- $U_j(0) = 0$, i.e., without consumption, satisfaction is zero.

[1]Since the single period problem can be extended to a multiple period problem with temporally coupled constraints, we solve the problem for a single market period to demonstrate the performance of the proposed method in a more explicit manner.

We consider a piece-wise quadratic utility function for consumer j as follows [22], [23]:

$$U_j(p_j) = \begin{cases} \beta_j p_j - \frac{1}{2}\theta_j p_j^2 : & 0 \leq p_j \leq \frac{\beta_j}{\theta_j} \\ \frac{\beta_j^2}{2\theta_j} : & p_j \geq \frac{\beta_j}{\theta_j} \end{cases} \tag{5}$$

The utility function parameters β_j and θ_j are the private information of consumer j.

The cost function $C_i(p_i)$ of the generator owned by producer i is a quadratic convex function of power p_i [24] as

$$C_i(p_i) = a_i p_i^2 + b_i p_i + c_i \tag{6}$$

The cost function parameters a_i, b_i, and c_i are the private information of producer i. The coefficient of losses in (4) satisfies $0 \leq \varrho_i \leq a_i$.

B. Network Utilization Fees for P2P Transactions

A new network structure called an electrical structure of the network based on the electrical distance is used to calculate the network utilization fees. The electrical structure of the network has the same number of the node to node connections as in the topological structure [25]. The network owner provides the charging rate for network utilization in advance before the P2P negotiation starts. The network owner considers the capital cost recovery, cost of maintenance and modernization of power lines, taxes and policies, etc. to decide the rate for the network utilization. The detailed study of how the network owner decides the rate for using the network is beyond the scope of this paper.

Now, if consumer j buys the p_{ji} amount of power from producer i over the electrical distance of d_{ji}, the network fee for consumer j is estimated by

$$T(p_{ji}) = \gamma d_{ji} p_{ji} \tag{7}$$

The total network fee to be paid by consumer j is

$$TF_j = \sum_{i \in \mathcal{P}} \gamma d_{ji} p_{ji} \tag{8}$$

There are various approaches such as the Thevenin's impedance distance, mutual impedance distance, power transfer distance (PTD), Jacobian distance to estimate the electrical distance between nodes in the power system [26]. In this paper, we use PTD to estimate the electrical distances between producers and consumers, and PTD is used interchangeably with the electrical distance. PTD between two nodes indicates how much of the network's assets are used in facilitating a P2P transaction between two nodes. PTD is calculated using the *Power Transfer Distribution Factor* (PTDF). The PTDF accounts for the fraction of transacted power from one node to another node that flows over a given line $l \in \mathcal{L}$. The detail method to calculate PTDF is given in [27] and summarized in Appendix A. The PTD between consumer j and producer i is

$$d_{ji} = \sum_{l \in \mathcal{L}} \left| PTDF_{l,ji} \right|. \tag{9}$$

C. Welfare of Consumer and Producer

In the P2P market, consumers negotiate with each producer at the same time. It means each consumer can buy energy from different producers with various marginal costs, and for each trade, there is a different network utilization fee. Therefore, a consumer has different valuation or welfare for each trade. If consumer j trades energy with producer i in P2P manner, the welfare of consumer j is given by the utility of the demand p_{ji} minus the sum of the paid money for this energy and the network utilization fee to be paid for this trade. Mathematically,

$$w_{ji} = U_j(p_{ji}) - \lambda_i p_{ji} - T(p_{ji}) \tag{10}$$

where λ_i is the per unit price of energy from producer i. The total welfare of consumer is given by the sum of the welfare from all possible P2P trades, i.e., $\sum_{i \in \mathcal{P}} w_{ji}$, and can be expressed as

$$W_j = \sum_{i \in \mathcal{P}} U_j(p_{ji}) - \sum_{i \in \mathcal{P}} \lambda_i p_{ji} - TF_j \tag{11}$$

The welfare of producer i is modeled as

$$W_i = \lambda_i (p_i - \phi_i(p_i)) - C_i(p_i) \tag{12}$$

The first term in (12) indicates the revenue collected by selling energy to the consumers and the second term represents the corresponding generation cost.

D. Optimization Problem

In this paper, the P2P energy trading in smart grids is formulated as a social welfare maximization problem as follows:

$$\underset{\mathbf{P}, \mathbf{p_p}}{\arg\max} \quad \sum_{j \in \mathcal{C}} \sum_{i \in \mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i \in \mathcal{P}} C_i(p_i) \tag{13a}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{C}} p_{ji} = p_i - \phi_i(p_i), \quad \forall i \in \mathcal{P} \tag{13b}$$

$$\underline{p}_i \leq p_i \leq \overline{p}_i, \quad \forall i \in \mathcal{P} \tag{13c}$$

$$\underline{p}_j \leq \sum_{i \in \mathcal{P}} p_{ji} \leq \overline{p}_j, \quad \forall j \in \mathcal{C} \tag{13d}$$

where $\hat{U}_j(p_{ji}) = U_j(p_{ji}) - T(p_{ji})$; and $\mathbf{p_p} = [\, p_i \,]_{i \in \mathcal{P}}$. A balance between the supply and demand in the system is essential for a stable operation of the power grid. The constraint (13b) represents the power balance constraint for producer i, i.e., the total power demanded by consumers from producer i should match the total generation less the loss contribution of producer i. For the feasibility of (13), Assumption 1 must be satisfied.

Assumption 1: The demand from producer $i \in \mathcal{P}$ must be higher than its minimum generation capacity, i.e.,

$$\sum_{j \in \mathcal{C}} p_{ji} \geq \underline{p}_i - \phi_i(\underline{p}_i) \tag{14}$$

A dual variable corresponding (13b), i.e., λ_i represents the energy price of producer i. A power balance in the system is established when individual producer satisfies (13b). Hence, (13b) is a global constraint, and (13c) and (13d) are the local capacity constraints of each producer and consumer, respectively. The nonlinear equality constraint (13b) makes the optimization problem (13) non-convex, which is difficult to solve directly. However, the same optimal solution can be recovered by transforming the non-convex problem (13) into a strictly convex problem (15) by relaxing the non-convex equality constraint.

$$\arg\max_{\mathbf{P},\mathbf{p_p}} \sum_{j\in\mathcal{C}}\sum_{i\in\mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i\in\mathcal{P}} C_i(p_i) \tag{15a}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{C}} p_{ji} \le p_i - \phi_i(p_i), \quad \forall i \in \mathcal{P}$$
$$\text{(13c) and (13d).} \tag{15b}$$

Theorem 1: The transformed problem (15) has the same optimal solution as the original problem (13).

Proof of Theorem 1: The proof is given in Appendix B. ∎

III. Market Clearing Algorithm Design

The transformed problem (15) can be solved in a centralized fashion having a coordinator with all the information of agents in the market. But, the presence of a coordinator may breach the privacy of agents and affect the fairness in the market clearing process, which is undesirable. Hence, we propose a decentralized approach to solve (15), where each agent needs to solve its sub-problem locally with a limited amount of information from other agents. Due to the presence of spatially coupled constraints (13d) and (15), problem (15) cannot be solved directly. Firstly, problem (15) is decomposed into a series sub-problems based on the principle of dual decomposition [28] and the sub-problems are solved distributively. Since the electricity price is a crucial variable used by all agents in the market, it is used to realize the coordination among producers and consumers. The detailed methodology is explained in the following sections.

A. Decoupling Into Sub-Problems

Let us define a Lagrangian of transformed primal problem (15) by relaxing the spatially coupled constraint (13d) and (15b) as follows:

$$\begin{aligned}
&\mathscr{L}\Big(\mathbf{P}, \mathbf{p_p}, \Lambda, \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}\Big) \\
&= \sum_{j\in\mathcal{C}}\sum_{i\in\mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i\in\mathcal{P}} C_i(p_i) \\
&\quad + \sum_{j\in\mathcal{C}} \underline{\mu}_j\Bigg(\sum_{i\in\mathcal{P}} p_{ji} - \underline{p}_j\Bigg) + \sum_{j\in\mathcal{C}} \overline{\mu}_j\Bigg(\overline{p}_j - \sum_{i\in\mathcal{P}} p_{ji}\Bigg) \\
&\quad + \sum_{i\in\mathcal{P}} \lambda_i\Bigg(p_i - \phi_i(p_i) - \sum_{j\in\mathcal{C}} p_{ji}\Bigg)
\end{aligned} \tag{16}$$

where $\lambda_i \ge 0$ is the Lagrangian multiplier or dual variable for producer i corresponding to constraint (15b); $\underline{\mu}_j, \overline{\mu}_j \ge 0$ are Lagrangian multipliers for consumer j corresponding to (13d); and $\Lambda \triangleq [\lambda_i]_{i\in\mathcal{P}}$, $\overline{\boldsymbol{\mu}} \triangleq [\overline{\mu}_j]_{j\in\mathcal{C}}$ and $\underline{\boldsymbol{\mu}} \triangleq [\underline{\mu}_j]_{j\in\mathcal{C}}$ are vectors of Lagrangian multipliers. From an economic point of view, the dual variable λ_i represents the energy price of producer i to maintain the balance between supply and demand. The constraint in (13c) is not included in the Lagrangian equation as they are local constraints and can be treated as the boundaries of the feasible region of the local problems.

The supremum of the Lagrangian over the variables $\mathbf{P}$ and $\mathbf{p_p}$ gives a dual function as

$$\begin{aligned}
\mathscr{D}\Big(\Lambda, \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}\Big) &= \sup_{\mathbf{P},\mathbf{p_p}} \mathscr{L}\Big(\mathbf{P}, \mathbf{p_p}, \Lambda, \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}\Big) \\
&= \sum_{i\in\mathcal{P}}\sum_{j\in\mathcal{C}} \mathscr{G}_{ji}\Big(\lambda_i, \underline{\mu}_j, \overline{\mu}_j\Big) + \sum_{i\in\mathcal{P}} \mathscr{H}_i(\lambda_i) \\
&\quad + \sum_{j\in\mathcal{C}}\Big(\overline{\mu}_j\overline{p}_j - \underline{\mu}_j\underline{p}_j\Big)
\end{aligned} \tag{17}$$

where $\mathscr{H}_i(\lambda_i)$ is the sub-problem to be solved by producer i and $\mathscr{G}_{ji}(\lambda_i, \underline{\mu}_j, \overline{\mu}_j)$ is the subproblem to be solved by consumer j to trade energy with producer i. The sub-problems are defined as follows

$$\mathscr{H}_i(\lambda_i) \triangleq \arg\max_{\underline{p}_i\le p_i\le\overline{p}_i} \big[\lambda_i(p_i - \phi_i(p_i)) - C_i(p_i)\big] \tag{18}$$

$$\mathscr{G}_{ji}\Big(\lambda_i, \underline{\mu}_j, \overline{\mu}_j\Big) \triangleq \arg\max_{0\le p_{ji}\le\overline{p}_j} \big[\tilde{U}_j(p_{ji}) - \lambda_i p_{ji}\big] \tag{19}$$

where $\tilde{U}_j(p_{ji}) \triangleq \hat{U}_j(p_{ji}) + (\underline{\mu}_j - \overline{\mu}_j)p_{ji}$. The social welfare maximization concurrently maximizes the individual welfare of the consumers and producers. Now, the dual problem is defined as

$$\begin{aligned}
&\arg\min_{\Lambda,\underline{\boldsymbol{\mu}},\overline{\boldsymbol{\mu}}} \ \mathscr{D}\Big(\Lambda, \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}}\Big) \\
&\text{s.t.} \quad \lambda_i, \underline{\mu}_j, \overline{\mu}_j \ge 0, \ \forall i \in \mathcal{P}, \ \forall j \in \mathcal{C}.
\end{aligned} \tag{20}$$

Theorem 2: The transformed problem (15) holds the strong duality.

Proof of Theorem 2: The proof is given in Appendix C. ∎

The dual problem (20) can be solved iteratively by deploying the sub-gradient projection method. The Lagrangian multipliers are updated in the opposite direction to the sub-gradient of the dual function as

$$\underline{\mu}_j^{k+1} = \Big[\underline{\mu}_j^k - \kappa\nabla_{\underline{\mu}_j}\mathscr{D}\Big]^+ \tag{21a}$$

$$\overline{\mu}_j^{k+1} = \Big[\overline{\mu}_j^k - \kappa\nabla_{\overline{\mu}_j}\mathscr{D}\Big]^+ \tag{21b}$$

$$\lambda_i^{k+1} = \Big[\lambda_i^k - \kappa\nabla_{\lambda_i}\mathscr{D}\Big]^+ \tag{21c}$$

Each consumer updates $\underline{\mu}_j$ and $\overline{\mu}_j$ in each iteration. These values are not shared with any other entities in the market. But, each producer updates λ_i in each iteration and shares with consumers in the market.

B. Sub-Problem Solution

At each iteration k, given the value of dual variable, i.e., price λ_i^k, each consumer obtains energy demand from producer i, $\hat{p}_{ji}^k$ by locally (independently) solving the sub-problem (19),

$$\hat{p}_{ji}^k = \Big[u_j^{-1}\Big(\lambda_i^k\Big)\Big]_0^{\overline{p}_j}, \ \forall j \in \mathcal{C} \tag{22}$$

where $u_j(p_{ji}) = \frac{\partial\tilde{U}_j(p_{ji})}{\partial p_{ji}}$.

Algorithm 1 Price Update by Producer i

Require: Termination criteria ϵ_λ
Ensure: Energy price for P2P energy trading λ_i
1. Assign the initial value of energy price λ_i^0;

while ($|\lambda_i^{k+1} - \lambda_i^k| > \epsilon_\lambda$) **do**
2. Broadcast energy price λ_i^k to all consumers;
3. Receive all consumers' demand $\hat{p}_{ji}^k$ for the given price;
4. Update production $\hat{p}_i^k$ by (23);
5. Update the energy price by (24);

end while

Algorithm 2 Demand Update by Consumer j

Require: Energy price $\Lambda^k = [\lambda_1^k, \lambda_2^k, \cdots, \lambda_{n_p}^k]$
Ensure: Power consumption p_j^k
while ($|\underline{\mu}_j^{k+1} - \underline{\mu}_j^k| > \epsilon_\mu$ & $|\overline{\mu}_j^{k+1} - \overline{\mu}_j^k| > \epsilon_\mu$) **do**
1. Receive energy price λ_i^k from producer i;
2. Update demand $\hat{p}_{ji}^k$ from producer i by (22);
3. Broadcast $\hat{p}_{ji}^k$ to corresponding producer i;
4. Update Lagrangian multipliers $\underline{\mu}_j^{k+1}$ and $\overline{\mu}_j^{k+1}$ by (25);

end while

Similarly, each producer i independently solves (18) for a given λ_i^k to determine the energy production. Define $v_i(p_i) = \frac{\partial C_i(p_i)}{\partial p_i}(1 - \frac{\partial \phi_i(p_i)}{\partial p_i})^{-1}$, then

$$\hat{p}_i^k = \left[v_i^{-1}\left(\lambda_i^k\right)\right]_{\underline{p}_i}^{\overline{p}_i}, \ \forall i \in \mathcal{P} \tag{23}$$

The update rule for the dual variable λ_i becomes

$$\lambda_i^{k+1} = \left[\lambda_i^k - \kappa\left(\left(\hat{p}_i^k - \phi_i\left(\hat{p}_i^k\right)\right) - \sum_{j\in\mathcal{C}} \hat{p}_{ji}^k\right)\right]^+ \tag{24}$$

The update rules for the Lagrangian multipliers $\underline{\mu}_j$ and $\overline{\mu}_j$ are:

$$\underline{\mu}_j^{k+1} = \left[\underline{\mu}_j^k - \kappa\left(\sum_{i\in\mathcal{P}} \hat{p}_{ji}^k - \underline{p}_j\right)\right]^+ \tag{25a}$$

$$\overline{\mu}_j^{k+1} = \left[\overline{\mu}_j^k - \kappa\left(\overline{p}_j - \sum_{i\in\mathcal{P}} \hat{p}_{ji}^k\right)\right]^+ \tag{25b}$$

In order to ensure the convergence, the value of κ in (23) and (24) should be sufficiently small such that $0 < \kappa < 2/L$, where L is the Lipschitz constant [29] for the dual function:

$$\|\nabla\mathscr{D}(\Omega_1) - \nabla\mathscr{D}(\Omega_2)\|_F \le L\|(\Omega_1) - \Omega_2)\|_F \tag{26}$$

where $\Omega = (\boldsymbol{\Lambda}, \underline{\boldsymbol{\mu}}, \overline{\boldsymbol{\mu}})$ is the single Lagrangian multiplier, and $\|\cdot\|_F$ is the matrix Frobenius norm. The stopping criterion are $|\lambda_i^{k+1} - \lambda_i^k| < \epsilon_\lambda$, $|\underline{\mu}_j^{k+1} - \underline{\mu}_j^k| < \epsilon_\mu$, and $|\overline{\mu}_j^{k+1} - \overline{\mu}_j^k| < \epsilon_\mu$. The process of the price update by producer i and the demand update by consumer j is summarized in Algorithm 1 and Algorithm 2, respectively. The proposed algorithms are executed through two-way communications between producers and consumers. Energy price and energy demand are the two pieces information need to be exchanged between each producer and consumer during P2P trading negotiation. Fig. 2 illustrates the interaction between producers and consumers during P2P trading negotiation.

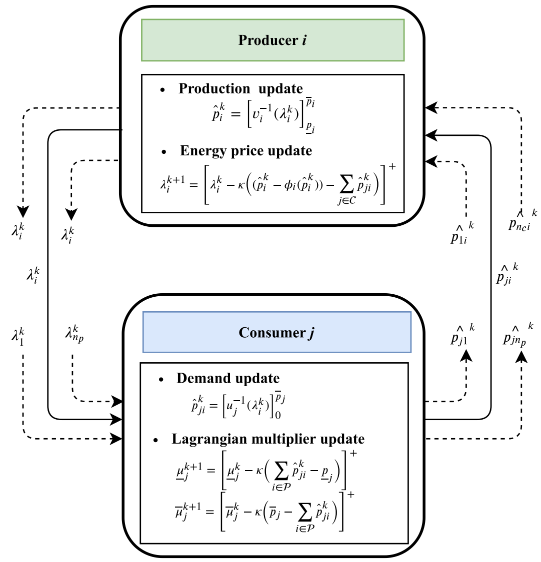

Fig. 2. Illustration of information exchange between producers and consumers during P2P trading negotiation.

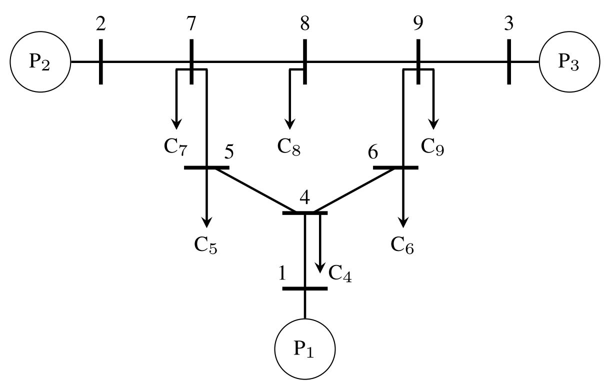

Fig. 3. IEEE 9-bus system for simulation studies.

IV. Results and Discussion

This section presents numerical case studies to show the feasibility and effectiveness of the proposed method for the P2P energy trading. For numerical case studies, we consider an IEEE 9-bus system with three producers and six consumers, as shown in Fig 3. The parameters for consumers and producers in the IEEE 9-bus system are taken from [15], [30] and given in Table I. Based on the data and the proposed algorithm, all the simulations are conducted in the MATLAB 2016a environment with an Intel Xeon CPU E5-1630 v4@3.70 GHz, 16 GB RAM. The step-size for the Lagrangian multiplier update κ is chosen as 0.005. The required tolerances for termination are set to $\epsilon_\lambda = 0.001$ and $\epsilon_\mu = 0.001$. The initial values are set as $\lambda_i^0 = \frac{\partial C_i}{\partial p_i}|_{p_i = \underline{p}_i}$, $\forall i \in \mathcal{P}$ and the charging rate for network usage is assumed to be $\gamma = 0.2$ \$/MWh per electrical distance unit. Following four cases are considered for the numerical studies.

- *Case 1:* P2P trading without losses and network fees.

TABLE I
CONSUMERS AND PRODUCERS PARAMETERS FOR IEEE 9-BUS SYSTEM

Producer	a_i (\$/MWh^2)	b_i (\$/MWh)	$\underline{p}_i$ (MW)	$\overline{p}_i$ (MW)	ϱ_i
P_1	0.0080	2.25	10	350	0.0005
P_2	0.0062	4.20	20	290	0.0007
P_3	0.0075	3.25	15	400	0.0004
Consumer	θ_j (\$/MWh^2)	β_j (\$/MWh)	$\underline{p}_j$ (MW)	$\overline{p}_j$ (MW)	
C_4	0.0720	8.25	60	150	-
C_5	0.0660	7.90	50	100	-
C_6	0.0700	7.55	90	145	-
C_7	0.0550	8.00	60	140	-
C_8	0.0750	7.75	50	150	-
C_9	0.0450	8.05	70	170	-

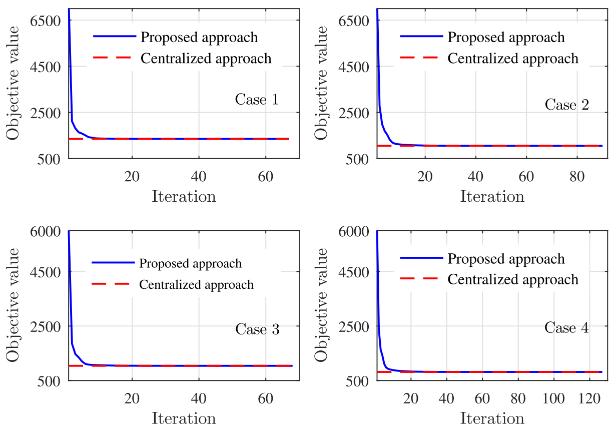

Fig. 4. Evolution of objective value in different cases.

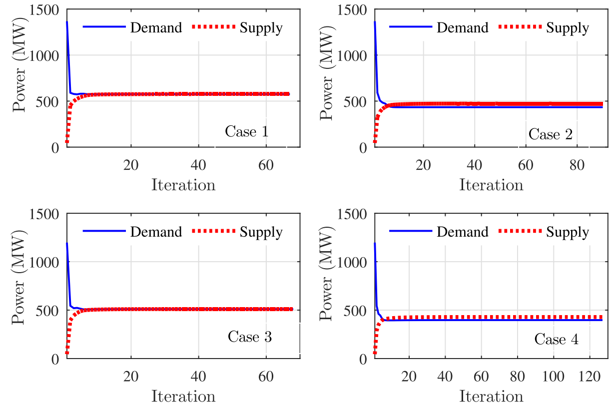

Fig. 5. Evolution of total demand and supply in different cases.

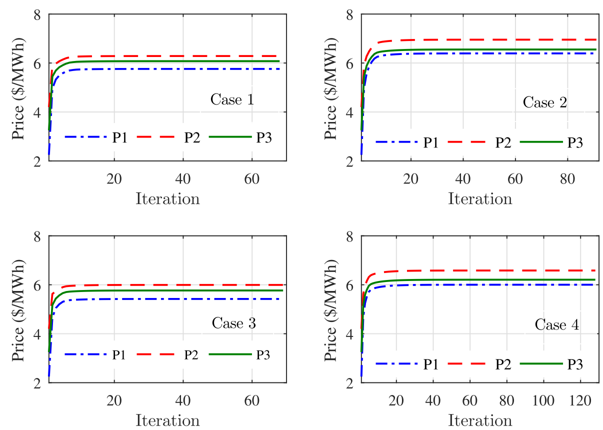

Fig. 6. Producers prices update during P2P negotiation in different cases.

- *Case 2:* P2P trading with losses.
- *Case 3:* P2P trading with network fees.
- *Case 4:* P2P trading with both losses and network fees.

It is worth mentioning that the results from the centralized approach are used as a benchmark to validate the results from the proposed decentralized approach. The centralized approach is implemented using the interior-point method in *Gurobi 8.1.1*.

Fig. 4 shows the development of the objective value for different cases using the proposed decentralized approach. It can be seen that the optimal objective value obtained from the proposed decentralized algorithm in all cases match the global optimal objective values obtained from the centralized approach. However, the convergence speed of the algorithm differs among cases. The proposed decentralized algorithm for P2P energy trading satisfies the termination conditions when the number of iterations $k = 67$ in *Case 1* and the results converge. Similarly, the algorithm converges in $k = 90$, $k = 68$, and $k = 127$ iterations in *Case 2*, *Case 3*, and *Case 4*, respectively. Fig. 4 also shows that the total objective value decreases when power losses and network utilization fees are considered in the P2P trading as compared with that of *Case 1*. The evolution of the total supply and demand in different cases are shown in Fig. 5. The total supply meets the total demand, and thus, the power balance condition is satisfied gradually in *Case 1* and *Case 3*. Unlike in *Case 1* and *Case 3*, the total supply does not meet the total demand in *Case 2* and *Case 4* because power losses are considered in these two cases. Once the power losses are taken into account, the total supply should meet the sum of the demand and losses. The mismatch between the total supply and the total demand is the total losses in the system. Fig. 6 shows the evolution of the producers prices in the above mentioned four cases. The evolution of the supply of producers in different cases are shown in Fig. 7. It shows that producers update their production decision in response to the evolution in their prices.

Table IV shows the output of producers obtained from the proposed decentralized approach and centralized approach in different cases. The results from two approaches are comparable. Fig. 8 shows the evolution of primal residuals given by $\|\mathbf{P}^k - \mathbf{P}^*\|_2$ in different cases, where $\mathbf{P}^k$ is the demand matrix $\mathbf{P}$ at the k^{th} iteration; $\mathbf{P}^*$ is the global optimal solution obtained by using the centralized approach; and $\|\cdot\|_2$ is the Euclidean norm. The primal residual or Euclidean norm indicates how close is the optimal solution obtained from the proposed decentralized algorithm with the optimal global solution obtained from the centralized approach. The solution is considered to be more accurate if the value of the Euclidean norm is small. The value of the Euclidean norm is less than 0.01 for all cases considered in this paper, which is negligible. Hence, the solution from the proposed method converges to the global optimum solution despite the individuals behave in a greedy manner.

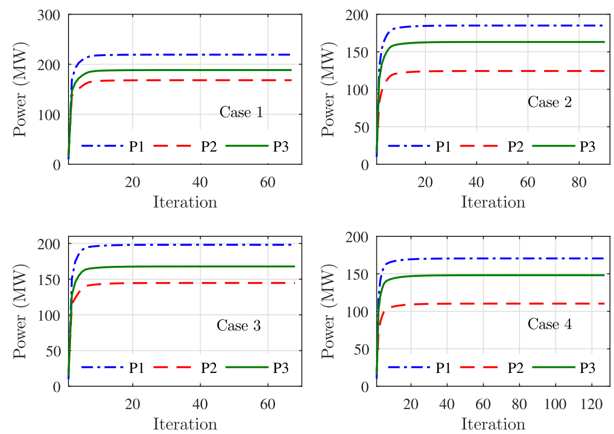

Fig. 7. Supply adjustment of producers in response to prices in different cases.

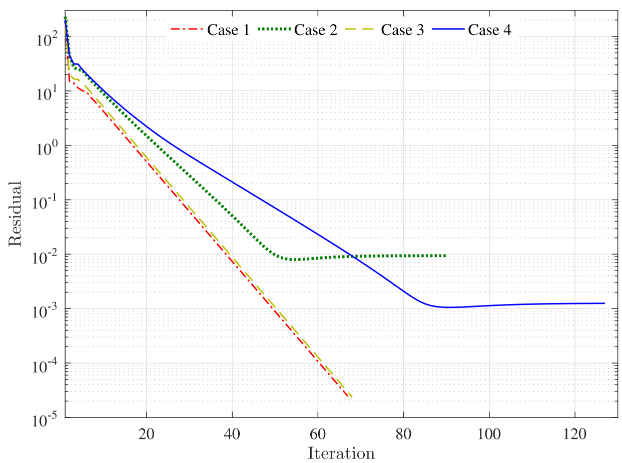

Fig. 8. Evolution of primal residuals $\|\mathbf{P}^k - \mathbf{P}^*\|_2$ in different cases.

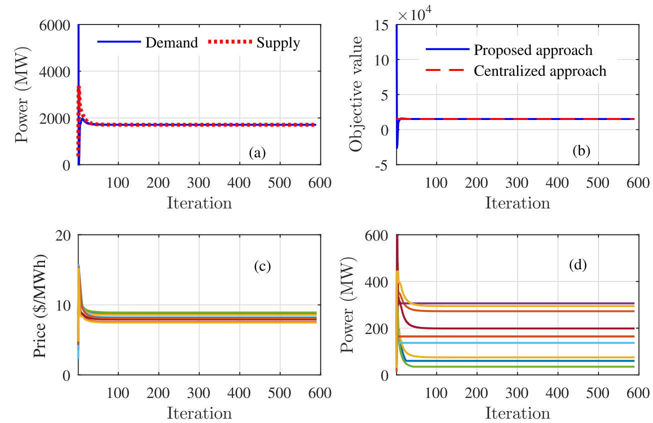

Fig. 9. Convergence characteristics of the proposed method in IEEE 39 bus system without considering losses and network fees (*Case 1*).

TABLE II
PTD FOR IEEE 9-BUS SYSTEM IN FIG. 3

Producer	Consumer					
	C_4	C_5	C_6	C_7	C_8	C_9
P_1	1.00	2.50	2.54	3.72	4.00	3.77
P_2	3.72	2.95	4.00	1.00	2.42	3.51
P_3	3.77	4.00	3.00	3.51	2.59	1.00

TABLE III
PRICE OF PRODUCERS FOR P2P ENERGY TRADING

Producer	Energy price ($/MWh)			
	Case 1	Case 2	Case 3	Case 4
P_1	5.7586	6.3935	5.4205	6.0017
P_2	6.2853	6.9535	5.9940	6.5830
P_3	6.0765	6.5523	5.7671	6.2071

Table II shows the PTD for different trading pairs for P2P energy transactions in the IEEE 9-bus system shown in Fig. 3. These PTD are used to calculate the network utilization fee for every possible P2P transaction. The energy prices of different producers for the P2P energy trading in different cases are shown in Table III. The electricity prices in *Case 2* are higher compared with those of *Case 1* due to power losses. The producers set higher prices because they have to bear the cost of losses from the revenue of the actual energy sold. Table V shows the trading amount between different pairs in the above mentioned four different cases. It is obvious that the amount of energy transacted between the trading pair is less in *Case 2* as compared with the same in *Case 1* because of the higher prices in *Case 2*. But, when the network utilization fee is considered in the P2P trading, the amount of energy transacted between the trading pairs depends not only on the energy price but also on the network fee. The trading of consumer C_9 with different producers in *Case 3* is chosen to explain the effect of the network utilization fee on P2P trading decisions. Consumer C_9 is buying the largest amount of power from producer P_3 despite the price offered by producer P_1 is the cheapest. Such alteration in trading decisions is because of the electrical distance between the trading pairs. It is clear from Table II that producer P_3 is electrically near to C_9 as compared to P_1, i.e., PTD between P_3 and C_9 is 1.00 whereas PTD between P_1 and C_9 is 3.77. Hence, consumer C_9 has to pay more network fees if it buys more power from P_1. So it prefers to buy from P_3. Similar observations can be made for other consumers too. The combined effects of power losses and network fees on P2P trading decisions can be observed in *Case 4*. It is clear that when we consider network utilization fees in P2P energy trading, the decision of consumers depends on the prices offered by producers as well as the electrical distance from producers.

We also apply the proposed algorithm on the IEEE 39-bus system with 18 consumers and 10 producers to demonstrate the scalability. The parameters of producers and consumers are taken from [14], [15]. Fig. 9 shows the evolution of the different parameters, i.e., the total supply and total demand, objective value, producers' prices, and supply, using the proposed method in the IEEE 39-bus system without considering losses and network fees. The total supply equates the total demand, and the power balance condition is being satisfied when the number of iterations $k = 589$ for the same level of accuracy as in the case of the IEEE 9-bus system. Fig. 9(c) and 9(d) show an evolution of producers' prices update and supply decisions update, respectively. These figures consist of 10 curves in each as there are 10 producers in the system. As the prices converge, the objective value, i.e., social welfare, converges to its optimal value, as shown in Fig. 9(b). Fig. 10

TABLE IV
COMPARISON OF MW OUTPUT OF PRODUCERS FROM CENTRALIZED AND PROPOSED DECENTRALIZED APPROACH

Method	Case 1			Case 2			Case 3			Case 4		
	P_1	P_2	P_3	P_1	P_2	P_3	P_1	P_2	P_3	P_1	P_2	P_3
Centralized	219.291	168.171	188.436	185.046	124.413	163.149	198.157	144.677	167.809	170.520	110.243	148.109
Proposed	219.291	168.171	188.436	185.032	124.400	163.144	198.157	144.677	167.809	170.517	110.243	148.109

TABLE V
DEMAND OF INDIVIDUAL CONSUMER FROM DIFFERENT PRODUCERS USING PROPOSED METHOD

Consumer	Case 1			Case 2			Case 3			Case 4		
	P_1	P_2	P_3	P_1	P_2	P_3	P_1	P_2	P_3	P_1	P_2	P_3
C_4	34.602	27.284	30.187	25.785	18.008	23.579	36.521	20.993	24.013	28.728	13.091	18.181
C_5	32.445	24.465	27.628	22.826	14.342	20.419	29.994	19.952	20.195	22.607	12.446	14.947
C_6	34.022	26.498	29.480	33.423	25.424	31.154	36.208	23.845	29.947	35.573	23.098	31.329
C_7	40.752	31.176	34.972	29.209	19.028	26.321	33.263	32.836	27.843	22.796	22.127	19.843
C_8	26.551	19.529	22.313	19.861	12.395	17.744	20.393	16.952	19.526	17.510	13.964	18.525
C_9	50.919	39.215	43.855	36.181	24.368	33.281	41.679	30.099	46.286	28.764	17.010	36.509

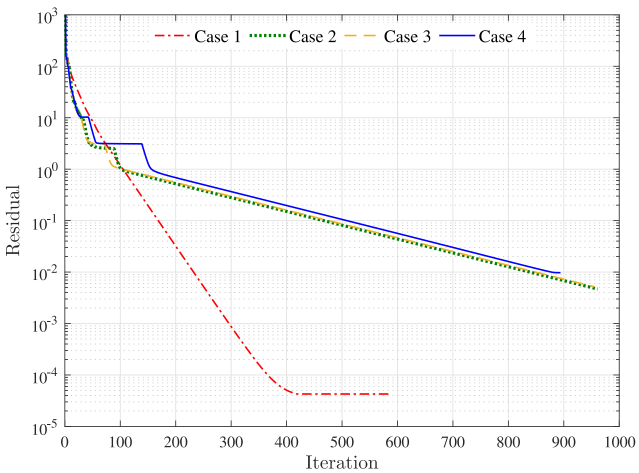

Fig. 10. Evolution of primal residuals $\|\mathbf{P}^k - \mathbf{P}^*\|_2$ in different cases for IEEE 39-bus system.

shows the evolution of the primal residuals. It demonstrates the convergence of the proposed decentralized algorithm in the IEEE 39-bus system. The optimal objective value from the proposed approach is the same as the one obtained from the centralized method. The small value of the primal residual and the matching of the optimal objective value are evidence of the convergence of the proposed algorithm. It is obvious that the extent of the computations being performed by each agent is determined by the number of iterations required for convergence. However, in the proposed algorithm, the computational burden is fairly shared among all agents in the system. Hence, the proposed decentralized algorithm for market clearing in the P2P energy trading is scalable in terms of computational burden and convergence.

V. CONCLUSION

In this paper, we presented a fully decentralized market clearing mechanism for the P2P energy trading. A P2P energy trading is formulated as an aggregated welfare maximization problem with consideration of grid usage aspect in P2P energy transactions. The original non-convex problem is transformed into a convex problem by relaxing the non-convex equality constraint under mild assumptions. The electricity prices and generation/demand are adjusted to maximize the social welfare and to achieve the balance between supply and demand in the market. The proposed method is applied to the IEEE 9-bus system and IEEE 39-bus system. The convergence and scalability of the proposed method are verified via numerical results. It is found that when we consider network fees in the P2P energy trading, the decision of consumers depends on the prices offered by producers as well as the electrical distance from producers. In the future, we plan to extend the P2P energy trading framework proposed in this paper to include network constraints using an optimal power flow.

APPENDIX A
PTDF CALCULATION METHOD

The PTDF matrix is calculated as

$$\mathbf{PTDF} = \mathbf{H_r B_r^{-1}} \tag{27}$$

where $\mathbf{H_r}$ is the sub-matrix of matrix $\mathbf{H}$ obtained by deleting the column corresponding to the slack node, and $\mathbf{B_r}$ is the sub-matrix of matrix B obtained by removing the row and column corresponding to the slack node. The matrices $\mathbf{H}$ and $\mathbf{B}$ are defined as

$$\mathbf{B}_{mn} = -\frac{1}{x_{mn}} \text{ for } m \neq n, \ \mathbf{B}_{mm} = \sum_{m \neq n} \frac{1}{x_{mn}} \tag{28}$$

$$\mathbf{H}_{lm} = -\mathbf{H}_{ln} = \frac{1}{x_{mn}}, \mathbf{H}_{lr} = \sum_{m \neq n} \frac{1}{x_{mn}} \text{ for } r \neq n, m \tag{29}$$

where indices m, n, and r represent nodes. The PTD between node m and node n denoted as d_{mn} is calculated by

$$d_{mn} = \sum_{l \in \mathcal{L}} \left| PTDF_{l,mn} \right|. \tag{30}$$

APPENDIX B
PROOF OF THEOREM 1

The Lagrangian of transformed problem (15) is

$$\begin{aligned}\mathscr{L} &= \sum_{j \in \mathcal{C}} \sum_{i \in \mathcal{P}} \hat{U}_j(p_{ji}) - \sum_{i \in \mathcal{P}} C_i(p_i) \\ &\quad + \sum_{i \in \mathcal{P}} \lambda_i \left(p_i - \varrho_i p_i^2 - \sum_{j \in \mathcal{C}} p_{ji} \right) + \sum_{i \in \mathcal{P}} \underline{\vartheta}_i \left(p_i - \underline{p}_i \right)\end{aligned}$$

$$+\sum_{i\in\mathcal{P}}\overline{\vartheta}\left(\overline{p}_i-p_i\right)+\sum_{j\in\mathcal{C}}\underline{\mu}_j\left(\sum_{i\in\mathcal{P}}p_{ji}-\underline{p}_j\right)$$
$$+\sum_{j\in\mathcal{C}}\overline{\mu}_j\left(\overline{p}_j-\sum_{i\in\mathcal{P}}p_{ji}\right) \quad (31)$$

The KKT conditions of (15) are given below in (32).

$$\frac{\partial\mathscr{L}}{\partial p_i}=-\frac{\partial C_i(p_i)}{\partial p_i}+\lambda_i(1-2\varrho_i p_i)-\overline{\vartheta}_i+\underline{\vartheta}_i=0;\ \forall i\in\mathcal{P} \quad (32a)$$
$$\frac{\partial\mathscr{L}}{\partial p_{ji}}=\frac{\partial\hat{U}_j(p_{ji})}{\partial p_{ji}}-\lambda_i-\overline{\mu}_i+\underline{\mu}_i=0;\ \forall j\in\mathcal{C} \quad (32b)$$
$$\lambda_i\left(\sum_{j\in\mathcal{C}}p_{ji}-p_i+\varrho_i p_i^2\right)=0,\ \lambda_i\geq 0;\ \forall i\in\mathcal{P} \quad (32c)$$
$$\overline{\vartheta}_i(\overline{p}_i-p_i)=0,\ \overline{\vartheta}_i\geq 0;\ \forall i\in\mathcal{P} \quad (32d)$$
$$\underline{\vartheta}_i(p_i-\underline{p}_i)=0,\ \underline{\vartheta}_i\geq 0;\ \forall i\in\mathcal{P} \quad (32e)$$
$$\overline{\mu}_j(\overline{p}_j-p_j)=0,\ \overline{\mu}_j\geq 0;\ \forall j\in\mathcal{C} \quad (32f)$$
$$\underline{\mu}_j(p_j-\underline{p}_j)=0,\ \underline{\mu}_j\geq 0;\ \forall j\in\mathcal{C} \quad (32g)$$

As per the KKT conditions (32c), $\lambda_i\geq 0$ for all $i\in\mathcal{P}$. If $\lambda_i>0$, (15b) is strictly satisfied by the optimal solution due to the KKT condtion (32c).

Since $\overline{p}_i>\underline{p}_i$ for all $i\in\mathcal{P}$, either $\overline{\vartheta}_i$ or $\underline{\vartheta}_i$ equals to zero due to the KKT conditions (32d) and (32e). Assume $\lambda_i=0$ for some $i\in\mathcal{P}$. Then, $\underline{\vartheta}_i$ should be positive to satisfy the KKT condition (32a), as $\frac{\partial C_i(p_i)}{\partial p_i}>0$ (strictly positive) for all $i\in\mathcal{P}$. This means λ_i can be equal to 0 only when $p_i=\underline{p}_i$. Then, the KKT condition (32c) implies either (33a) or (33b).

$$\sum_{j\in\mathcal{C}}p_{ji}<\underline{p}_i-\phi_i\left(\underline{p}_i\right) \quad (33a)$$
$$\sum_{j\in\mathcal{C}}p_{ji}=\underline{p}_i-\phi_i\left(\underline{p}_i\right) \quad (33b)$$

As per Assumption 1, (33a) is a contradiction and hence, (33b) is the only possible condition. Therefore, the optimal solution which satisfies the KKT conditions strictly satisfies (15b). Hence, the two problems (13) and (15) are equivalent.

Appendix C
Proof of Theorem 2

Let $\Psi_i\geq\varrho_i p_i^2>0$ for all $i\in\mathcal{P}$ be a decision variable of (15) such that

$$\sum_{j\in\mathcal{C}}p_{ji}=p_i-\Psi_i;\ \ \forall i\in\mathcal{P} \quad (34a)$$
$$\left(\frac{\Psi_i+1}{2}\right)^2\geq\left(\frac{\Psi_i-1}{2}\right)^2+\left(\sqrt{\varrho_i}p_i\right)^2;\forall i\in\mathcal{P} \quad (34b)$$

where (34a) is a linear constraint and (34b) is a convex second-order cone constraint. Accordingly, (34) can replace the power balance constraint (15b) in (15). Therefore, (15) is a convex (second-order cone) program which holds a strong duality.

References

[1] M. Khorasany, Y. Mishra, and G. Ledwich, "A decentralized bilateral energy trading system for peer-to-peer electricity markets," *IEEE Trans. Ind. Electron.*, vol. 67, no. 6, pp. 4646–4657, Jun. 2020.

[2] T. Baroche, F. Moret, and P. Pinson, "Prosumer markets: A unified formulation," in *Proc. IEEE Milan PowerTech*, Milan, Italy, Jun. 2019, pp. 1–6.

[3] J. Guerrero, A. C. Chapman, and G. Verbič, "Decentralized P2P energy trading under network constraints in a low-voltage network," *IEEE Trans. Smart Grid*, vol. 10, no. 5, pp. 5163–5173, Sep. 2019.

[4] M. Khorasany, Y. Mishra, and G. Ledwich, "Design of auction-based approach for market clearing in peer-to-peer market platform," *J. Eng.*, vol. 18, no. 18, pp. 4813–4818, Jul. 2019.

[5] A. Paudel, K. Chaudhari, C. Long, and H. B. Gooi, "Peer-to-peer energy trading in a prosumer-based community microgrid: A game-theoretic model," *IEEE Trans. Ind. Electron.*, vol. 66, no. 8, pp. 6087–6097, Aug. 2019.

[6] T. Baroche, P. Pinson, R. L. G. Latimier, and H. B. Ahmed, "Exogenous cost allocation in peer-to-peer electricity markets," *IEEE Trans. Power Syst.*, vol. 34, no. 4, pp. 2553–2564, Jul. 2019.

[7] K. Anoh, S. Maharjan, A. Ikpehai, Y. Zhang, and B. Adebisi, "Energy peer-to-peer trading in virtual microgrids in smart grids: A game-theoretic approach," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1264–1275, Mar. 2020.

[8] K. Zhang, S. Troitzsch, S. Hanif, and T. Hamacher, "Coordinated market design for peer-to-peer energy trade and ancillary services in distribution grids," *IEEE Trans. Smart Grid*, early access, Jan. 13, 2020, doi: 10.1109/TSG.2020.2966216.

[9] T. Chen and W. Su, "Indirect customer-to-customer energy trading with reinforcement learning," *IEEE Trans. Smart Grid*, vol. 10, no. 4, pp. 4338–4348, Jul. 2019.

[10] T. Morstyn, A. Teytelboym, and M. D. Mcculloch, "Bilateral contract networks for peer-to-peer energy trading," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 2026–2035, Mar. 2019.

[11] W. Tushar *et al.*, "Grid influenced peer-to-peer energy trading," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1407–1418, Mar. 2020.

[12] Z. Zhang, R. Li, and F. Li, "A novel peer-to-peer local electricity market for joint trading of energy and uncertainty," *IEEE Trans. Smart Grid*, vol. 11, no. 2, pp. 1205–1215, Mar. 2020.

[13] W. Liu, F. Wen, and D. Qi, "Intraday residential demand response scheme based on peer-to-peer energy trading," *IEEE Trans. Ind. Informat.*, vol. 16, no. 3, pp. 1823–1835, Mar. 2020.

[14] N. Rahbari-Asr, U. Ojha, Z. Zhang, and M. Chow, "Incremental welfare consensus algorithm for cooperative distributed generation/demand response in smart grid," *IEEE Trans. Smart Grid*, vol. 5, no. 6, pp. 2836–2845, Nov. 2014.

[15] C. Zhao, J. He, P. Cheng, and J. Chen, "Consensus-based energy management in smart grid with transmission losses and directed communication," *IEEE Trans. Smart Grid*, vol. 8, no. 5, pp. 2049–2061, Sep. 2017.

[16] A. Paudel, M. Khorasany, and H. B. Gooi, "Decentralized local energy trading in microgrids with voltage management," *IEEE Trans. Ind. Informat.*, early access, Mar. 12, 2020, doi: 10.1109/TII.2020.2980160.

[17] Y. Sun, X. Wu, J. Wang, D. Hou, and S. Wang, "Power compensation of network losses in a microgrid with bess by distributed consensus algorithm," *IEEE Trans. Syst., Man, Cybern., Syst.*, early access, Feb. 6, 2020, doi: 10.1109/TSMC.2020.2968605.

[18] J. Kim and Y. Dvorkin, "A P2P-dominant distribution system architecture," *IEEE Trans. Power Syst.*, early access, Dec. 23, 2019, doi: 10.1109/TPWRS.2019.2961330.

[19] A. I. Nikolaidis, C. A. Charalambous, and P. Mancarella, "A graph-based loss allocation framework for transactive energy markets in unbalanced radial distribution networks," *IEEE Trans. Power Syst.*, vol. 34, no. 5, pp. 4109–4118, Sep. 2019.

[20] J. Guerrero, A. C. Chapman, and G. Verbic, "Trading arrangements and cost allocation in P2P energy markets on low-voltage networks," in *Proc. IEEE Power Energy Soc. Gen. Meeting (PESGM)*, Atlanta, GA, USA, Aug. 2019, pp. 1–5.

[21] S. A.-H. Soliman and A.-A. H. Mantawy, *Modern Optimization Techniques With Applications in Electric Power Systems*. New York, NY, USA: Springer, 2011.

[22] P. Samadi, A. H. Mohsenian-Rad, R. Schober, V. W. S. Wong, and J. Jatskevich, "Optimal real-time pricing algorithm based on utility maximization for smart grid," in *Proc. 1st IEEE Int. Conf. Smart Grid Commun.*, Gaithersburg, MD, USA, Oct. 2010, pp. 415–420.

[23] S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing, and T. Başar, "Demand response management in the smart grid in a large population regime," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 189–199, Jan. 2016.

[24] J. J. Grainger and W. D. Stevenson, *Power Systems Analysis*. New York, NY, USA: McGraw-Hill, 1997.

[25] S. Poudel, Z. Ni, and W. Sun, "Electrical distance approach for searching vulnerable branches during contingencies," *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 3373–3382, Jul. 2018.

[26] P. Cuffe and A. Keane, "Visualizing the electrical structure of power systems," *IEEE Syst. J.*, vol. 11, no. 3, pp. 1810–1821, Sep. 2017.

[27] A. J. Wood, B. F. Wollenberg, and G. B. Sheblé, *Power Generation, Operation, and Control*. Hoboken, NJ, USA: Wiley, 2013.

[28] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, Jan. 2011.

[29] D. P. Bertsekas, "Nonlinear programming," *J. Oper. Res. Soc.*, vol. 48, no. 3, p. 334, 1997.

[30] L. Ding, L. Y. Wang, G. Y. Yin, W. X. Zheng, and Q. Han, "Distributed energy management for smart grids with an event-triggered communication scheme," *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 5, pp. 1950–1961, Sep. 2019.

Amrit Paudel (Graduate Student Member, IEEE) received the B.E. degree in electrical and electronic engineering from Pokhara University, Nepal, in 2012, the M.E. degree in energy engineering from the Asian Institute of Technology, Bangkok, Thailand, in 2016, and the Ph.D. degree in electrical and electronic engineering from Nanyang Technological University, Singapore, in 2020. His current research interests include microgrid energy management systems, peer-to-peer energy trading, and distribution-level electricity market.

L. P. M. I. Sampath (Member, IEEE) received the B.Sc. degree in electrical engineering from the University of Moratuwa, Sri Lanka, in 2014, and the Ph.D. degree in electrical engineering from the Interdisciplinary Graduate School, Nanyang Technological University, Singapore, in 2020, where he is currently a Research Fellow with the School of Electrical and Electronic Engineering. His research interests include convex optimization, modeling and optimization under renewable energy-based uncertainties, optimal power flow, and power system scheduling.

Jiawei Yang (Graduate Student Member, IEEE) received the B.E. degree in electrical engineering and automation from Wuhan University, China, in 2018, and the M.E. degree in electrical and electronic engineering from Nanyang Technological University, Singapore, in 2020, where he is currently pursuing the Ph.D. degree in electrical and electronic engineering. His current research interests include peer-to-peer energy trading and blockchain application for energy trading.

Hoay Beng Gooi (Life Senior Member, IEEE) received the B.S. degree in electrical engineering from National Taiwan University in 1978, the M.S. degree in electrical engineering from the University of New Brunswick in 1980, and the Ph.D. degree in electrical engineering from Ohio State University in 1983. He is an Associate Professor with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. His current research interests include microgrid energy management systems dealing with storage, electricity market, and spinning reserve.